Dimensioning CBS Parameters

What is the worst-case response time $R_{aperiodic}$ of an aperiodic task whose computation requirement is $C_{aperiodic}$?

Case 1: $C_{aperiodic} = k Q_{CBS}$, $k \in \mathbb{N}$ (natural numbers)

As we discussed in class, the aperiodic job experiences maximum interference. CBS guarantees that despite this, it gets at least Q_{CBS} time every period. Due to interference, the time that CBS gets might be pushed as late as possible. Hence, in this case,

$R_{case\,1}$

$$\underbrace{R_{aperiodic} = k \cdot T_{CBS} = \frac{k \cdot Q_{CBS}}{U_{CBS}} = \frac{C_{aperiodic}}{U_{CBS}}}_{R_{case\,1}}$$

we represent in terms of U_{CBS} because while Q_{CBS} & T_{CBS} are unknowns (& we are trying to dimension them), U_{CBS} is known

Case 2: $C_{aperiodic} = k Q_{CBS} + k'$, $k \in \mathbb{N}$ and $0 < k' < Q_{CBS}$

For the first part of $C_{aperiodic}$ that is kQ_{CBS} units long, the response time is $k T_{CBS}$ (like in Case 1). For the second part, the response time is $(k+1)T_{CBS} - \Delta$, where Δ is also the difference between Q_{CBS} & k'. Thus,

$$\begin{aligned}
R_{aperiodic} &= (k+1)\,T_{CBS} - \Delta \\
&= (k+1)\,T_{CBS} - (Q_{CBS} - k') \\
&= (k+1)\,T_{CBS} - Q_{CBS} + C_{aperiodic} - kQ_{CBS} \\
&= (k+1)(T_{CBS} - Q_{CBS}) + C_{aperiodic} \\
&= \left\lceil \frac{C_{aperiodic}}{Q_{CBS}} \right\rceil (T_{CBS} - Q_{CBS}) + C_{aperiodic} \quad \text{(Eq 6.8 in textbook)} \\
&= (k+1)\left(\frac{Q_{CBS}}{U_{CBS}} - Q_{CBS}\right) + C_{aperiodic} \\
&= (k+1)\,Q_{CBS}\left(\frac{1 - U_{CBS}}{U_{CBS}}\right) + C_{aperiodic}
\end{aligned}$$

Special case of Case 2, where Q_{CBS} is <u>slightly</u> smaller than $C_{aperiodic}$:

$$\begin{aligned}
Q_{CBS} &= C_{aperiodic} - \varepsilon \\
C_{aperiodic} &= Q_{CBS} + \varepsilon \quad \Big\}\ \begin{matrix} k = 1 \\ k' = \varepsilon \end{matrix} \ \text{in Case 2}
\end{aligned}$$

$$R_{aperiodic} = (k+1)(C_{aperiodic} - \varepsilon)\frac{(1 - U_{CBS})}{U_{CBS}} + C_{aperiodic}$$

let suppose, $\varepsilon = C_{aperiodic}/1000$ & since $k = 1$

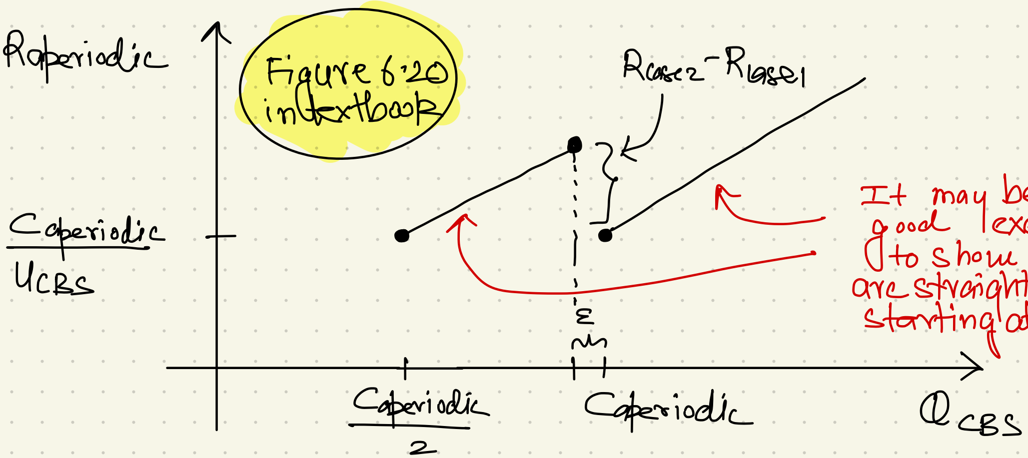
$$R_{aperiodic} = 2\,(0.999)\,C_{aperiodic}\frac{(1 - U_{CBS})}{U_{CBS}} + C_{aperiodic}$$

$$= \frac{C_{aperiodic}}{U_{CBS}} \left(1.998 - 1.998\,U_{CBS} + U_{CBS}\right)$$

$$= \frac{C_{aperiodic}}{U_{CBS}} \left(1.998 - 0.998\,U_{CBS}\right)$$

$$= R_{case1} \left(1.998 - 0.998\,U_{CBS}\right) \longrightarrow R_{case2}$$

$$R_{case2} - R_{case1} = R_{case1}\,(0.998)(1 - U_{CBS})$$



It may be a good exercise to show why these are straight lines starting at $\frac{C_{aperiodic}}{U_{CBS}}$

Lets go to Eq 6.8 again.

$$R_{aperiodic} = \left\lceil \frac{C_{aperiodic}}{Q_{CBS}} \right\rceil (T_{CBS} - Q_{CBS}) + C_{aperiodic}$$

Since $\frac{C_{aperiodic}}{Q_{CBS}} < \left\lceil \frac{C_{aperiodic}}{Q_{CBS}} \right\rceil < \frac{C_{aperiodic}}{Q_{CBS}} + 1$

$$R_{aperiodic} > \frac{C_{aperiodic}}{Q_{CBS}} (T_{CBS} - Q_{CBS}) + C_{aperiodic}$$

$$= \frac{C_{aperiodic}}{Q_{CBS}} T_{CBS} \qquad \text{Eq 6.11}$$

$$R_{aperiodic} < \left(\frac{C_{aperiodic}}{Q_{CBS}} T_{CBS} \right) + \left(T_{CBS} - Q_{CBS} \right) \qquad \text{Eq 6.10}$$

Overhead. Assume due to overheads, we really get only $Q_{CBS} - \gamma$ worth of budget, where γ = context switch delay

Thus, from 6.10, $$R^{max}_{aperiodic} = \left(\frac{C_{aperiodic}\, T_{CBS}}{Q_{CBS} - \gamma}\right) + T_{CBS} - Q_{CBS} + \gamma$$

If $f(x)$ = prob. that $C_{aperiodic} = x$,

then $$\bar{R}^{max}_{aperiodic} = \int_0^\infty \left[\frac{x\, T_{CBS}}{Q_{CBS} - \gamma} + T_{CBS} - Q_{CBS} + \gamma\right] f(x)\, dx$$

denotes mean

$$= T_{CBS} - Q_{CBS} + \gamma + \frac{T_{CBS}}{Q_{CBS} - \gamma} \int_0^\infty x f(x)\, dx$$

why?

$$= T_{CBS} - Q_{CBS} + \gamma + \frac{T_{CBS}}{Q_{CBS} - \gamma} \bar{C}_{aperiodic} \quad \text{Eq 6.13}$$

Differentiating $\bar{R}^{max}_{aperiodic}$ with respect to T_{CBS} will give us T_{CBS} for which $\bar{R}^{max}_{aperiodic}$ is min/max.

when is it a min & when is it a max?

Follow Eq 6.14 & 6.15 in textbook.

We get eventually,

$$T_{CBS} = \frac{1}{U_{CBS}} \left(\gamma + \sqrt{\frac{\gamma \bar{C}_{aperiodic}}{1 - U_{CBS}}}\right).$$