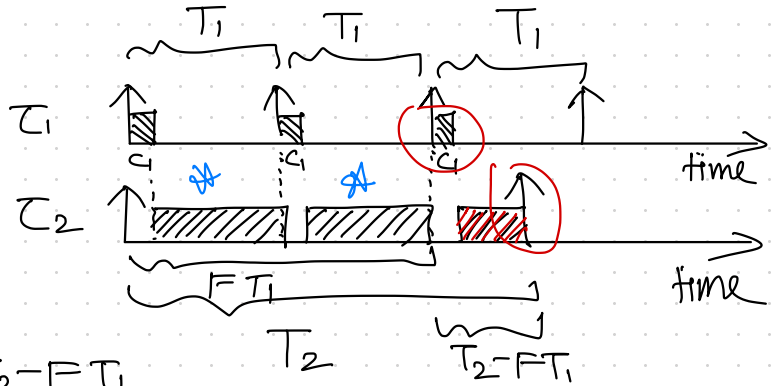


$$\tau = \{\tau_1, \tau_2\}$$

$$\tau_1 = (T_1, C_1)$$

$$\tau_2 = (T_2, C_2)$$

$$T_1 < T_2$$



Case a $C_1 < T_2 - FT_1$

where $F = \lfloor \frac{T_2}{T_1} \rfloor$

$F = 2$

$F = 3?$

$$(T_1 - C_1) + (T_1 - C_1) + (T_2 - FT_1 - C_1)$$

$$C_2 \leq F(T_1 - C_1) + (T_2 - FT_1 - C_1) \Rightarrow \text{RM schedulability}$$

$$= \cancel{FT_1} - C_1 F + T_2 - \cancel{FT_1} - C_1$$

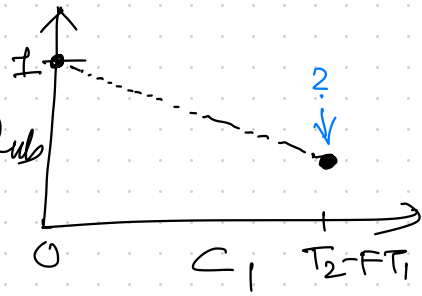
$$= T_2 - C_1(1 + F)$$

$$U_{ub} = \frac{C_1}{T_1} + \frac{C_2}{T_2}$$

$$= \frac{C_1}{T_1} + \frac{T_2 - C_1(1 + F)}{T_2}$$

$$= 1 + C_1 \left[\frac{1}{T_1} - \frac{1 + F}{T_2} \right]$$

$$= 1 + \frac{C_1}{T_2} \left[\frac{T_2}{T_1} - 1 - F \right]$$



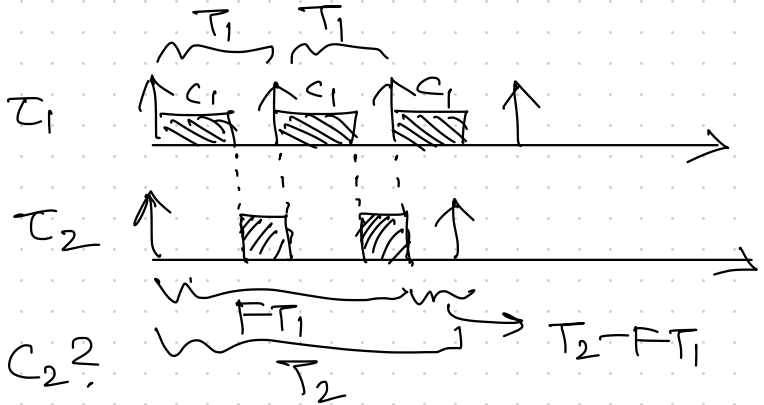
$$U_{ub} = 1 + \frac{(T_2 - FT_1)(\frac{T_2}{T_1} - 1 - F)}{T_2}$$

$$= 1 + \cancel{\frac{C_1 T_2}{T_2 T_1}} - \cancel{\frac{C_1}{T_2}} - \frac{C_1 F}{T_2} - \frac{C_1 F T_2}{T_2 T_1} + \frac{C_1 T_2}{T_2} + \frac{C_1^2}{T_2}$$

$$= 1 - 2F + \frac{C_1 T_2}{T_2} + \frac{C_1^2}{T_2}$$

Case b

$$C_1 \geq T_2 - FT_1$$



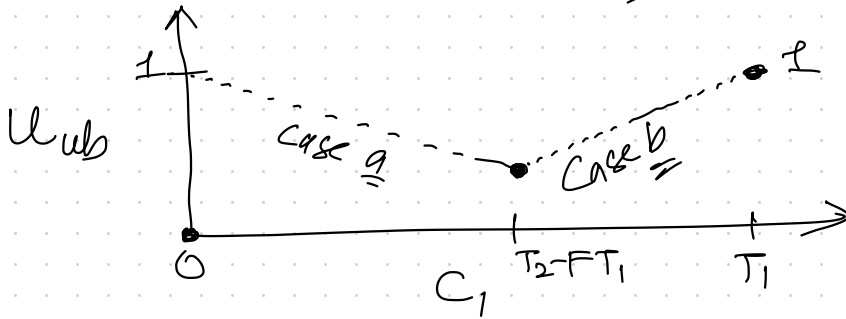
What's the max C_2^2 ?

$$C_2 \leq F(T_1 - C_1) + 0$$

$$U_{ub} = \frac{C_1}{T_1} + \frac{F(T_1 - C_1)}{T_2}$$

$$= \frac{FT_1}{T_2} + \frac{C_1}{T_1} - \frac{FC_1}{T_2}$$

$$= \frac{FT_1}{T_2} + \underbrace{\frac{C_1}{T_2} \left[\frac{T_2}{T_1} - F \right]}_{\geq 0} = \frac{T}{C_1 = T_1}$$



$$\begin{aligned} U_{ub} &= \frac{FT_1}{T_2} + \frac{T_2 - FT_1}{T_2} \left(\frac{T_2}{T_1} - F \right) \\ &= F \frac{T_1}{T_2} + \left(1 - F \frac{T_1}{T_2} \right) \frac{T_2}{T_1} \left(1 - F \frac{T_1}{T_2} \right) \\ &= F \frac{T_1}{T_2} + \frac{T_2}{T_1} \left(1 - F \frac{T_1}{T_2} \right)^2 \end{aligned}$$

$$\begin{aligned} &\vdots \\ \text{Eq. (4.5)} &= \frac{T_1}{T_2} \left[F + \left(\frac{T_2}{T_1} - F \right)^2 \right] \end{aligned}$$