Lecture #7 : Periodic Task Scheduling
(DM Schedulability Analysis)

$\tau = \{\tau_1, \tau_2, \dots, \tau_n\}$ Preemption

Each $\tau_i = (C_i, T_i, D_i, \phi_i)$

Assume: $D_1 < D_2 < \dots < D_n$

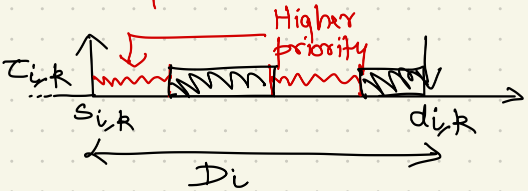
DM: $prio(\tau_1) > prio(\tau_2) > \dots$

Goal: DM schedules every task τ_i without any deadline misses

$\Downarrow$

$\forall i, k$: $\tau_{i,k}$ does not miss its deadline.

"Interference" I

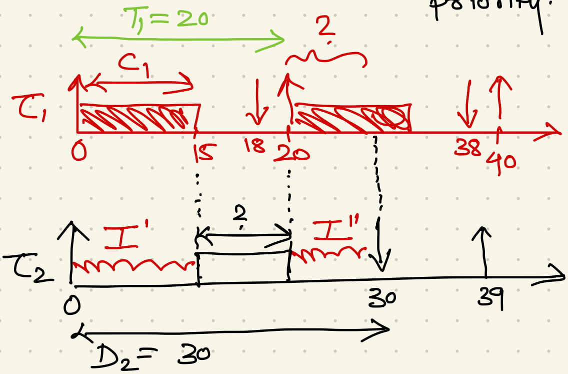


$D_i - I \geq C_i$

$C_i + \underline{\underline{I}} \leq D_i$

	T	D	C
τ_1	20	18	15
τ_2	39	30	5
τ_3	100	90	8

$D_1 < D_2$
$\downarrow$
higher priority.



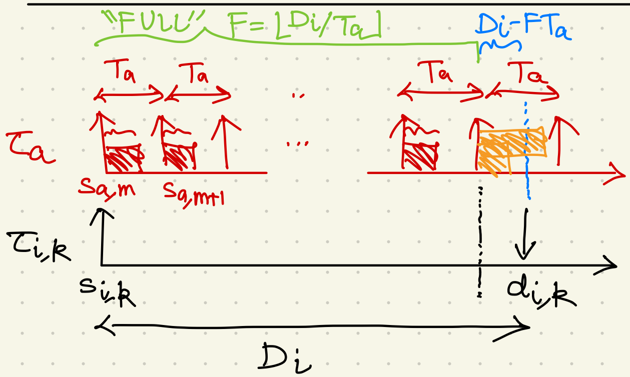
$C_2 + I \leq D_2$

$I' = C_1 = 15$

$I'' = D_2 - T_1 = 30 - 20 = 10$

$C_2 + I' + I'' \leq D_2$

$5 + 15 + 10 \leq 30$? YES



$C_i + \underline{\underline{I^a_{i,k}}} \leq D_i$

$\underline{\underline{I^{a,red}_{i,k}}}$ = FC_a

$\underline{\underline{I^{a,orange}_{i,k}}}$ → C_a, $D_i - FT_a$ → $\min(D_i - FT_a, C_a)$

$C_i + I^{a,red}_{i,k} + I^{a,orange}_{i,k} \leq D_i$

$$\boxed{\forall i,k : \quad C_i + \sum_{\substack{\text{all higher}\\ \text{priority tasks}\\ \tau_a}} \left(I^{a,\text{red}}_{i,k} + I^{a,\text{orange}}_{i,k} \right) \leq D_i}$$

$$0 \to \infty$$

$$\underline{\underline{\forall i}} : \quad C_i + \max_{\forall k} \sum_{\forall a < i} \underline{I^{a,\text{red}}_{i,k}} + \underline{I^{a,\text{orange}}_{i,k}} \leq D_i$$

$$\forall i : \quad C_i + \sum_{\forall a < i} \underbrace{\max_{\forall k}}_{\text{redundant}} \underbrace{\left\lfloor \frac{D_i}{T_a} \right\rfloor C_a + \min\left(D_i - \left\lfloor \frac{D_i}{T_a} \right\rfloor T_a, C_a \right)}_{\text{independent of } k} \leq D_i$$

$$\forall i : \quad C_i + \sum_{\forall a < i} \left(\left\lfloor \frac{D_i}{T_a} \right\rfloor C_a + \min\left(D_i - \left\lfloor \frac{D_i}{T_a} \right\rfloor T_a, C_a \right) \right) \leq D_i$$

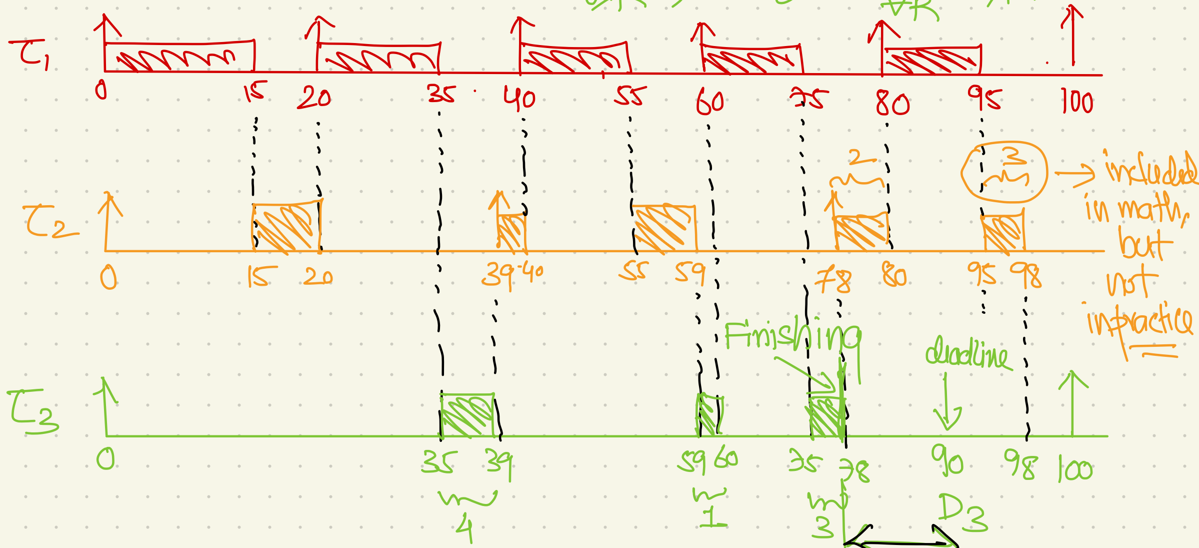
$$C_3 + \sum_{a < 3} \left(\left\lfloor D_3 / T_a \right\rfloor C_a + \min\left(D_3 - \left\lfloor D_3 / T_a \right\rfloor T_a, C_a \right) \right) \leq D_3$$

$$C_3 + \overbrace{\left\lfloor \frac{D_3}{T_1} \right\rfloor C_1 + \min\left(D_3 - \left\lfloor D_3 / T_1 \right\rfloor T_1, C_1 \right)}^{a=1} + \underbrace{\left\lfloor \frac{D_3}{T_2} \right\rfloor C_2 + \min\left(D_3 - \left\lfloor D_3 / T_2 \right\rfloor T_2, C_2 \right)}_{a=2} \leq D_3$$

$$8 + 4 * 15 + \min(90 - 4 * 20, 15) + 2 * 5 + \min(90 - 78, 5) = 8 + 60 + 10 + 10 + 5 = 93 \underset{\text{NO!}}{\leq} 90$$

Finishing time − starting time = R

$R_{i,k}$, $R_i = \max_{\forall k} R_{i,k}$



Old test : Sufficient but not necessary

$$C_i + I \leq D_i \Rightarrow C_i + \underline{\underline{I}} \leq R_i$$

I also depends on R_i

$\boxed{\text{Response-Time Analysis}}$

RTA

HW: Sec 4.5.1 & 4.5.2